Correlation between event multiplicity and elliptic flow parameter

Dai-Mei Zhou[1*], Yun Cheng[1,2,5], Yu-Liang Yan[3], Bao-Guo Dong[3], Ben-Hao Sa[1,3,4†], Laszlo P. Csernai[2,5‡]

[1] *Institute of Particle Physics, Huazhong Normal University,Wuhan, 430082 China*
[2] *Institute of Physics and Technology, University of Bergen, Allegaten 55, 5007 Bergen, Norway*
[3] *China Institute of Atomic Energy, P. O. Box 275 (18), Beijing, 102413 China*
[4] *CCAST (World Lab.), P. O. Box 8730 Beijing, 100080 China*
[5] *Frankfurt Institute for Advanced Studies, Johann Wolfgang Goethe University,*
Ruth-Moufang-Str. 1, 60438 Frankfurt am Main, Germany

The elliptic flow parameter (component), v_2, in the Fourier expansion of event-by-event charged particle multiplicity azimuthal distribution in the momentum space is studied by taking into account the multiplicity fluctuations. The correlations between charge multiplicity and impact parameter as well as between elliptic flow parameter and impact parameter (multiplicity) are investigated by both the parton and hadron cascade model PACIAE and a multiple phase transport model AMPT. A discussion is given for the event-wise average and particle-wise average in the definition of elliptic flow parameter.

PACS numbers: 25.75.Dw, 24.85.+p

I. INTRODUCTION

The charge particles emitted from the fireball created in relativistic nucleus-nucleus collisions exhibit transverse collective flow. This is represented by the elliptic flow parameter (v_2) and other harmonic parameters (v_n, $n=0, 1, 3, 4, ...$). Those parameters, as Fourier expansion coefficients of produced particle azimuthal distribution in the momentum space, are highly sensitive to the spatial geometry (eccentricity) of the created fireball (nuclear overlap region or interaction region).

The expected phase transition to Quark-Gluon-Plasma (QGP) should have a dramatic effect on those harmonic parameters. The consistency between experimental data of $v_2(p_T)$ ($v_2(y)$) at mid-rapidity and the corresponding hydrodynamic predictions is regarded as an evidence of the production of partonic matter in the ultra-relativistic nucleus-nucleus collisions [1, 2]. The elliptic flow of high p_T particles may be related to jet fragmentation and parton energy loss [3], which are usually not included in the hydrodynamic calculations. Such kind of hydrodynamic calculation [4] overestimates $v_2(p_T)$ in the $p_T \geq 1.5$ GeV/c region [5]. This is regarded as an evidence of the strongly coupled QGP formation in the relativistic nucleus-nucleus collisions together with the discovery of jet quenching [6]. So far a lot of experimental data have been published on the collective flow parameters [7, 8, 9]. Consequently microscopic transport model studies are also widely progressing [10, 11, 12, 13, 14] as well as the abundant hydrodynamic investigations.

According to two well known pioneering works in this field [15, 16], the usual study starts from the triple differential distribution

$$E\frac{d^3N}{d^3p} = \frac{1}{2\pi}\frac{d^2N}{p_T dy dp_T}[1+\sum_{n=1,...} 2v_n cos[n(\phi - \Psi_r)]], \quad (1)$$

where N is the particle multiplicity distribution, ϕ stands for the azimuthal angle of particle, and Ψ_r refers to the azimuthal angle of reaction plane in the momentum space. Then the n-th flow harmonics is defined as $v_n = \langle cos[n(\phi - \Psi_r)]\rangle$, where $\langle\rangle$ indicates an average over all particles in all events of a sample. For the distribution of all particles in the sample, the coefficients in Eq. (1) can be calculated as $v_1 = \langle p_x/p_T\rangle$ and $v_2 = \langle (p_x/p_T)^2 - (p_y/p_T)^2\rangle$, etc. This kind of average is widely accepted and will be indicated as particle-wise average hereafter.

The problem of the particle-wise average in a sample with wider multiplicity range is that it does not take the influence of multiplicity (hence impact parameter) in a single event of the sample into account. In [17] the elliptic flow and other harmonics have been re-derived starting from the invariant particle multiplicity distribution in the momentum space. It turned out that the harmonic parameters v_n (elliptic flow parameter v_2) is an event-wise average of $cos(n\phi)$

$$v_n^e = \langle\overline{cos(n\phi)}\rangle_{ev} \quad (n = 1, 2, ...), \quad (2)$$

where $\overline{cos(n\phi)}$ denotes the average of $cos(n\phi)$ over particles in a single event, $\langle...\rangle_{ev}$ means an average over events in a sample, and the superscript "e" stands for the event-wise average. It is an average of $cos(n\phi)$ first over particles in a single event then over the events in a sample. Here it has to mention that in theory if the beam direction and impact parameter vectors are fixed at the p_z and p_x axes, respectively, then the reaction plane is just the $p_x - p_z$ plane [15]. Therefore the reaction plane azimuthal angle (Ψ_r) in Eq. (1) introduced for the extraction of elliptic flow in experiments [16] is zero. Meanwhile, the

particle-wise average of v_n is also derived in [17]

$$v_n^p = \left\langle \overline{cos(n\phi)} N_{ev} \right\rangle_{ev} \; / \; \langle N_{ev} \rangle_{ev} . \qquad (3)$$

That is obviously different from event-wise average. Only if the $\overline{cos(n\phi)}$ is independent of event multiplicity N_{ev} (i. e. if the multiplicity plays no role in the average) the v_n^p reduces to v_n^e. In fact, the $\overline{cos(n\phi)}$ and N_{ev} correlate (even negatively correlate) with each other. This is because larger event multiplicity arises from more central collisions (larger overlap region between colliding nuclei) and the larger overlap region, in turn, results in less azimuthal asymmetry. The particle-wise average does not take the influence of event multiplicity into account, thus it is questionable from physics point of view. Of course, for a very narrow multiplicity bin studied, the particle-wise average is not very problematic. However for the wide multiplicity bin the correction is important. Here we do not take the influence of eccentricity fluctuation on v_2 into account, it may strengthen the importance of the correction.

In experiment, the reaction plane is different event by event. In order to extract the elliptic flow parameter one has to invoke a complex reaction plane identification method [16], the cumulant method [18], and the Lee-Yang zeroes method [19]. In all of these methods a quantity has to be first constructed event by event. This quantity is just the event plane in [16], the cumulant expansion of the weighted n-th transverse event-flow vector in [18], and a generating function in [19]. Then a corresponding average over measured events has to be taken. Therefore the experimental extraction of elliptic flow parameter is an event-wise average. In the recent paper [20] it has been pointed out that "Elliptic flow develops on an event-by-event basis. However, the experimental determination of v_2 demands averaging over events and thus over distributions of geometric shapes." This means again the experimental extraction of v_n is an event-wise average.

In the expansion process of a nucleus-nucleus collision, the initial spatial asymmetry of the created fireball evolves into an azimuthal asymmetry (v_n) in the transverse momentum distribution of produced charged particles [9]. This evolution process has to be described precisely by detailed dynamical models. Thus, in this work a parton and hadron cascade model PACIAE [21] and a multiple phase transport model AMPT (with string melting) [22] are both used in order to have cross checking. The observables of the impact parameter (b), charged particle multiplicity (N_{ch}), eccentricity (ϵ), and elliptic flow parameter (v_2) as well as other harmonic parameters (v_n) are always used to describe the physics of the initial spatial asymmetry, final momentum asymmetry, and their fluctuations and correlations. As first step, in this paper we only study the correlation between N_{ch} and b as well as the correlation between v_2 and b (N_{ch}). In addition, the event-wise average versus particle-wise average in calculating (extracting) the elliptic flow pa-

rameter raised in [17] is also studied. The eccentricity fluctuation may have extra effect on the v_2 parameter and its fluctuation besides the impact parameter (multiplicity) fluctuation, which will be studied in next step. Our studies are based on the multiplicity fluctuation at fixed impact parameter, and would apply also if the initial eccentricity would not fluctuate.
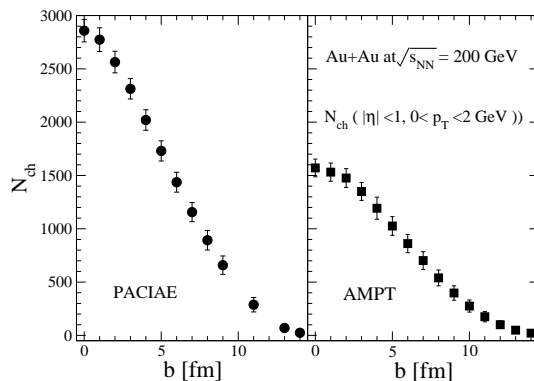

FIG. 1: Correlation between impact parameter b and charge multiplicity N_{ch} calculated by the PACIAE model (left panel) and the AMPT model (right panel) for the indicated pseudorapidity and p_T range. The error bars indicate the fluctuations of multiplicity at fixed impact parameter. Consequently a fixed event multiplicity may correspond to different impact parameters.

II. MODELS

The PACIAE model [21] is a parton and hadron cascade model and consists of four stages: parton initialization, parton evolution (rescattering), hadronization, and hadron evolution (rescattering).

In the first stage, a nucleus-nucleus collision is decomposed into nucleon-nucleon (NN) collisions according to the collision geometry:

- Both colliding nuclei settle down at the distance between two centers of colliding nuclei along the x-axis is equal to impact parameter b.

- Nucleons in the colliding nucleus are randomly distributed around the center of nucleus according to a Wood-Saxon distribution (r) and the 4π uniform distribution (θ and ϕ).

- The beam momentum is given to nucleons in the colliding nucleus. The nucleon Fermi motion is neglected.

- The NN collision is sampled randomly according to the spatial and momentum distributions, the straight line trajectories, and the NN interaction cross sections of the colliding pairs.

Then a NN collision is described by the PYTHIA model [23]. In the PYTHIA model

- The NN collision is decomposed into parton-parton collisions.

- A hard parton-parton collision is described by the lowest leading order perturbative QCD (LO-pQCD) parton-parton cross sections modified by the parton distribution function in a nucleon.

- The soft parton-parton collision is considered empirically.

- Because the initial- and final-state QCD radiations are considered in parton-parton interactions the consequence of a NN collision is partonic multijet configuration composed of di-quarks (anti-diquarks), quarks (anti-quarks), gluons, and a few hadronic remnants.

- The string fragmentation is performed. Then one obtains a final hadronic state for a NN collision (hence for a nucleus-nucleus collision).

However, in the PACIAE model the above string fragmentation is switched-off temporarily. The di-quarks (anti-diquarks) are broken randomly into quarks (anti-quarks). So, the consequence of a NN collision (hence a nucleus-nucleus collision) is a configuration of quarks, anti-quarks, gluons, and hadronic remnants. In addition, there are spectator nucleons for a nucleus-nucleus collision.

The partonic rescattering stage follows the initialization one. In this stage the rescattering among partons are considered by the $2 \to 2$ LO-pQCD differential cross sections [24]. The six elastic and three inelastic parton-parton scatterings are involved with their own differential cross sections [24]. We use the Monte Carlo method to simulate the parton rescattering until the parton-parton collisions cease.

In the hadronization stage the partons after rescattering are hadronized by either the string fragmentation scheme (Lund string fragmentation and/or independent fragmentation) [23] or the coalescence model [21].

The last stage of hadronic rescattering is described by the usual two-body collision method [25]. As such, the hadronic matter is evolved until the hadron-hadron collision pairs are exhausted.

The two models of PACIAE and AMPT with string melting are physically similar. They have similar model assumptions but have otherwise very little commons in detail. The main differences between them are as follows:

- AMPT is based on HIJING [3] instead of PYTHIA in PACIAE.

- In the AMPT model the hadrons from HIJING are fragmented into partons, while the spectator nucleons are kept survival. This parton initialization for a nucleus-nucleus collision is quite different from

the one in the PACIAE model, where parton initialization is realized by broking the strings before the string fragmentation. Thus, even HIJING itself is heavily based on PYTHIA, the initialized parton state is quite different between the PACIAE and AMPT models. There are much more partons in the AMPT model than in PACIAE.

- The Zhang parton cascade (ZPC) model [26] is employed to describe parton rescattering. In ZPC only elastic scatterings are considered with $gg \to gg$ cross section instead of all elastic interaction cross sections.

- Partons after rescattering are hadronized by the coalescence model only.

- The dynamics of the consequent hadronic matter is described by a relativistic transport (ART) model [27]. In ART the cross sections of hadron-hardon collisions and hadronic resonances are considered in detail. Therefore the hadronic rescattering is considered more carefully in AMPT than in PACIAE.

As mentioned in the beginning, the AMPT model has been used successfully to describe the elliptic flow parameter [10, 12] in Au+Au collisions at RHIC energy. The AMPT results given in this paper are calculated by the same code [12] with parameters adjusted to the v_2 data. However, the PACIAE results are calculated with the default parameters. Thus the two results are not matching each other very well. Nevertheless, that is irrelevant because we aim to explore the important physics mentioned above rather than to reproduce the experimental data. To demonstrate that physics and to estimate its quantitative importance we used two independent models, both having a realistic description for multiplicity fluctuations at the fixed impact parameter, at hand [28] to cross check the correlation between N_{ch} and b and the correlation between v_2 and b (N_{ch}).

III. RESULTS

The correlation between impact parameter b (in fm) and charged multiplicity N_{ch} from the PACIAE and AMPT calculations for Au+Au collisions at $\sqrt{s_{NN}}$=200 GeV are shown in Fig. 1. One sees in the left panel of Fig. 1 that N_{ch} is negatively correlated with b. A unit of "fm" change in impact parameter results in more than 100 charged particle change in multiplicity. An about 20% increase in multiplicity corresponds to about 2 fm decrease in impact parameter. Similar conclusions can be drawn from the AMPT results in right panel of Fig. 1.
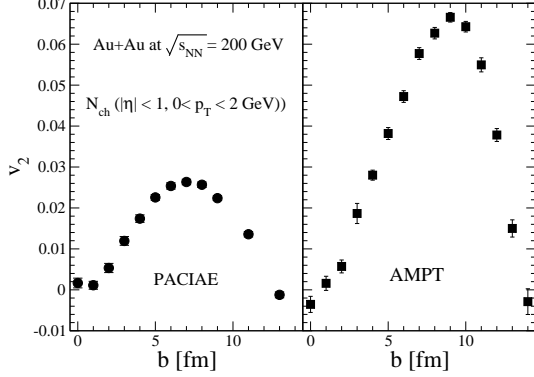
FIG. 2: Impact parameter dependence of integrated v_2 calculated by the PACIAE model (left panel) and the AMPT model (right panel). The error bars indicate the random fluctuations of v_2 at fixed impact parameter.
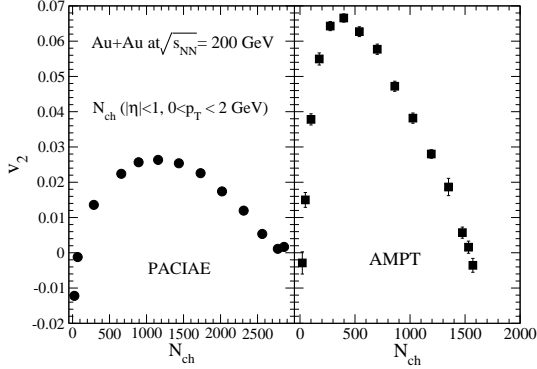

FIG. 3: Charged particle multiplicity dependence of integrated v_2 calculated by the PACIAE model (left panel) and the AMPT model (right panel). The error bars indicate the random fluctuations of v_2 at fixed charged particle multiplicity.

In Fig. 2 we give the integrated v_2 (event-wise average) as a function of impact parameter b calculated by the PACIAE and AMPT models for the Au+Au collisions at $\sqrt{s_{NN}}$=200 GeV. We see in left panel of Fig. 2 that there is a peak appearing at b nearly equal to the radius of colliding nucleus. That is a result of competition between the charge multiplicity and the impact parameter. In central Au+Au collision the charge multiplicity reaches maximum at zero impact parameter, where nuclear overlap region is nearly symmetric thus v_2 approaches zero [5, 9]. In the middle peripheral collision, although the charge multiplicity is going down, the v_2 is large because of the strong asymmetry of nuclear overlap region (large impact parameter). In the extra peripheral collision, the asymmetry of nuclear overlap region is very strong, while the v_2 is going down because the multiplicity is too low to generate pressure gradient. When the impact parameter is around the radius of colliding nucleus, the charge multiplicity is not so small, and the asymmetry of nuclear overlap region is considerably strong, so v_2 approaches its maximum. Similar situations can be seen in the AMPT results shown in the right panel of Fig. 2. These results

are consistent with the PHOBOS and PHENIX reports about the peak structure in v_2 as a function of the number of participant nucleons (N_{part}) in Au+Au collisions at $\sqrt{s_{NN}}$=200 GeV [5, 29]. Similarly, we give the integrated v_2 as a function of N_{ch} calculated by the PACIAE and AMPT models for the Au+Au collisions at $\sqrt{s_{NN}}$=200 GeV in Fig. 3.

In the left panel of Fig. 1 we know that a nearly 20% increase in multiplicity corresponds to about 2 fm decrease in impact parameter. This change in impact parameter, in turn, results in about 0.010 change in v_2 (see the sensitive region of $2 < b < 7$ in the left panel of Fig. 2). The similar, even stronger, conclusions can be drawn from the AMPT results in the right panels of Fig. 1 and 2. Thus, both the PACIAE and AMPT models, incorporating random multiplicity fluctuation, verify that a charge multiplicity bin, which includes e.g. 20% of maximum charged multiplicity, spans an impact parameter range of $\Delta b \sim 2$ fm. That results in a change of $\Delta v_2 \sim 0.010$ in the PACIAE calculations. Considering the maximum v_2 in the PACIAE model is just only 0.028, so the change of Δv_2 caused by the different impact parameters in the sample is about 30-40% of the maximum! This is a very significant change, which should not be underestimated!

In [17] the PACIAE model has been used to calculate the charged hadron $v_2(\eta)$ in 0-40% most central Au+Au collisions at $\sqrt{s_{NN}}$=200 GeV by the method of event-wise and particle-wise averages separately. The $v_2^e(\eta)$ is about 10% larger than $v_2^p(\eta)$. This means $\left\langle \overline{cos(n\phi)} N_{ev} \right\rangle_{ev}$ is smaller than $\left\langle \overline{cos(n\phi)} \right\rangle_{ev} \langle N_{ev} \rangle_{ev}$ and demonstrates the negative correlation between $\overline{cos(n\phi)}$ and N_{ev}. Here we use the AMPT model [22] to calculate $v_2^e(\eta)$ and $v_2^p(\eta)$ in 0-40% most central Au+Au collisions at $\sqrt{s_{NN}}$=200 GeV again. The AMPT results of $v_2^e(\eta)$ are nearly 20% larger than $v_2^p(\eta)$, as shown in Fig. 4.
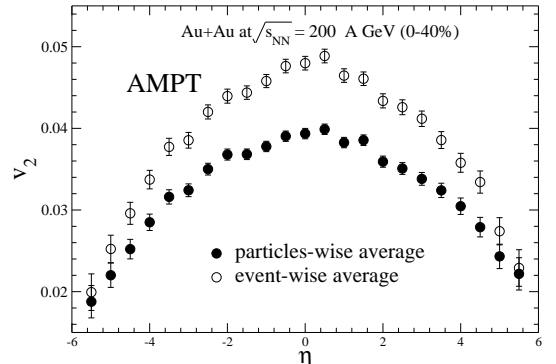

FIG. 4: v_2 as a function of η calculated by the AMPT model for $0-40\%$ most central Au+Au at $\sqrt{s_{NN}}$= 200 GeV, using the two methods of averaging described in the text.

IV. CONCLUSIONS

In summary, we have used the parton and hadron cascade model PACIAE and a multiple phase transport model AMPT with string melting to investigate the elliptic flow parameter v_2 by taking into account the event-by-event charge multiplicity fluctuations and to cross check the physics of initial spatial asymmetry, final momentum asymmetry, and their fluctuations and correlations. The correlations between the charge multiplicity and impact parameter as well as between the elliptic flow parameter and impact parameter (charge multiplicity) are calculated.

It turned out that the charge multiplicity is negatively correlated with impact parameter. The v_2 as a function of impact parameter first increases with increasing b, reaches its maximum at b close to the radius of colliding nucleus, and turns to decrease with increasing b at last. This is because the v_2 is determined by two driving forces, multiplicity and impact parameter.

The averaging procedure in the definition of elliptic flow parameter is reexamined by the AMPT calculations for $v_2^e(\eta)$ and $v_2^p(\eta)$ in 0-40% most central Au+Au collisions at $\sqrt{s_{NN}}$=200 GeV. The AMPT results of $v_2^e(\eta)$ are about 20% larger than $v_2^p(\eta)$, which are larger than the corresponding PACIAE results [17]. This emphasizes again the necessity of theoretically calculating the elliptic flow parameter v_2 by the event-wise average, in order to be consistent with the experimental extraction of v_2, which is really event-wise average [17, 20].

[1] M. Gyulassy, arXiv:nucl-th/0403032, and in "Structure and dynamics of elementary matter: Proceedings of the NATO Advanced Study Institute on Structure and Dynamics of Elementary Matter, Camyuva-Kemer (Antalya), Turkey, 22 September-2 October 2003, ed. by W. Greiner.

[2] L.P. Csernai and D. Röhrich, Phys. Lett. **B 458** 454 (1999).

[3] M. Gyulassy and X.-N. Wang, Comput. Phys. Commun. **83**, 307 (1994).

[4] P. F. Kolb, P. Huovinen, U. W. Heiz, and H. Heiselberg, Phys. Lett. B **500**, 232 (2001).

[5] B. B. Back, et al., Phobos collaboration, Phys. Rev. C **72**, 051901(R) (2005).

[6] W. A. Zajc, arXiv:0802.3552.

[7] B. I. Abelev, STAR collaboration, Phys. Rev. Lett. **99**, 112301 (2007), and references therein.

[8] S. S. Adler, et al., PHENIX collaboration, Phys. Rev. Lett. **99**, 052301 (2007), and references therein.

[9] B. Alver, et al., PHOBOS collaboration, Phys. Rev. Lett. **98**, 242302 (2007), and references therein.

[10] Bin Zhang, M. Gyulassy, and Che-Ming Ko, Phys. Lett. B **455**, 45 (1999).

[11] E. E. Zabrodin, C. Fuchs, L. V. Bravina, and A. Faessler, Phys. Lett. B **508**, 184 (2001).

[12] Lie-Wen Chen, V. Greco, Che-Ming Ko, and P. F. Kolb, Phys. Lett. B **605**, 95 (2005).

[13] Zhe Xu, C. Greiner, and H. Stoeker, Phys. Rev. Lett. **101**, 082902 (2008).

[14] Xianglei Zhu, M. Bleicher, and H. Stöcker, Phys. Rev. C **72**, 064911 (2005).

[15] S. A. Voloshin and Y. Zhang, Z. Phys. C **70**, 665 (1996).

[16] A. M. Poskanzer and S. A. Voloshin, Phys. Rev. C **58**, 1671 (1998).

[17] Xiao-Mei Li, Bao-Guo Dong, Yu-Liang Yan, Hai-Liang Ma, Dai-Mei Zhou and Ben-Hao Sa, to appear in Modern Phys. Lett. A; arXiv:nucl-th/0804.3655v3.

[18] N. Borghini, P. M. Dinh, and J.-Y. Ollitrault, Phys. Rev. C **63**, 054906 (2001).

[19] R. S. Bhalerao, N. Borghini, J.-Y. Ollitrault, Nucl. Phys. A **727**, 373 (2003).

[20] J. Casalderrey-Solana and U. A. Wiedemann, arXiv:0911.4400v1.

[21] Ben-Hao Sa, Xiao-Mei Li, Shou-Yang Hu, Shou-Ping Li, Jing Feng, and Dai-Mei Zhou, Phys. Rev. C **75**, 054912 (2007); Dai-Mei Zhou, Xiao-Mei Li, Bao-Guo Dong, and Ben-Hao Sa, Phys. Lett. B **638**, 461 (2006).

[22] Z. W. Lin, C. M. Ko, B. A. Li, B. Zhang, and S. Pal, Phys. Rev. C **72**, 064901 (2005).

[23] T. Söjstrand, S. Mrenna, and P. Skands, J. High Energy Phys. **JHEP05**, 026 (2006); hep-ph/0603175.

[24] B. L. Combridge, J. Kripfgang, and J. Ranft, Phys. Lett. B **70**, 234 (1977).

[25] Ben-Hao Sa and Tai An, Comput. Phys. Commun. **90**, 121 (1995); Tai An and Ben-Hao Sa, Comput. Phys. Commun. **116**, 353 (1999).

[26] B. Zhang, Comput. Phys. Commun. **109**, 193 (1998).

[27] B. A. Li and C. M. Ko, Phys. Rev. C **52**, 2037 (1995).

[28] We thank Prof. Lie-Wen Chen for the helps in running the AMPT model.

[29] M. Shimomura, PHENIX collaboration, arXiv:0908.0052v2 .